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TECHNICAL NOTE

An approximate solution of the convective heat transfer from an isothermal rotating cylinder

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An approximate solution is presented for the calculation of the convective heat transfer rates through a laminar boundary layer over a rotating circular cylinder in a fluid of unlimited extent. By using the appropriate velocity components in the energy equation, a solution was derived for the average Nusselt number as $\mathrm{Nu} = 0.6366(\mathrm{Re\ Pr})^{1/2}$. The solution compares well with the available experimental data.

Keywords: forced convection; rotating cylinder; rotating flow; boundary layer

Introduction

Convective heat transfer from a spinning circular cylinder in still air is important in many engineering applications such as shafts, spinning projectiles, drying of paper on rollers in the paper industry, and cooling reentry space vehicles. As a consequence, there exist several experimental investigations for the problem in the literature; e.g., Dropkin and Carmi (1957), Kays and Bjorklund (1958), and Jones et al. (1988) among others.

The complexity of the structure of the spinning fluid motion around the rotating cylinder has drawn the attention of several researchers to study this problem theoretically (Daskalakis 1993). However, the work in this paper is focused on obtaining an analytical solution for the convective heat transfer over an isothermal spinning circular cylinder. Free convection effects were not considered here.

Theory

Consider a circular cylinder of radius a placed in an infinite stationary fluid medium. The cylinder is rotated about its axis at an angular velocity Ω. The molecular attraction between fluid particles causes the fluid immediately next to the surface of the spinning cylinder to rotate with it in a laminar concentric circular stream.

A two-dimensional (2-D) r, θ system of polar coordinates is chosen and the velocity components of the main rotating flow are (Schlichting 1979).

$$V_r = 0 \tag{1}$$

and

$$V_\theta = \Omega a^2/r \tag{2}$$

The origin of the (r, θ) coordinates is any point along the centerline of the rotating cylinder. The above velocity components are particular solutions of the steady-state Navier–Stokes equation for the case of an inviscid incompressible flow.

The spinning motion of the fluid near the cylinder surface causes the formation of a thin boundary layer of thickness δ. This assumption was proved experimentally by Kays and Bjorklund (1958). The thickness of the spinning boundary layer is assumed uniform all around the circumference of the cylinder without separation, as was demonstrated by Prandtl and Tietjens (1934). The heat transfer between the fluid and the isothermal surface of the cylinder takes place within the laminar boundary layer. If the heat conduction in the angular direction (i.e., $\partial^2 T/\partial\theta^2$) is small in comparison with the heat conduction in the radial direction (i.e., $\partial^2 T/\partial r^2$), then the steady-state energy conservation equation with negligible natural convection, and in the absence of internal heat source, becomes

$$V_r \frac{\partial T}{\partial r} + \frac{V_\theta}{r}\frac{\partial T}{\partial \theta} = \alpha\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) \tag{3}$$

The right-hand side of the equation above is approximated on the order of magnitude basis by using the boundary-layer assumption mentioned above. Because $(\partial^2 T/\partial r^2)$ is $0(1/\delta^2)$, and $(\partial T/r\partial r)$ is $0(1/a\delta)$, the latter being normally neglected compared with the former for $a \gg \delta$.

We introduce a new variable y in the direction normal to the cylinder surface so that $y = 0$ at $r = a$ defined as

$$(r-a)/a = y/a \ll 1 \tag{4}$$

The thin boundary-layer assumption allows us to say

$$\frac{V_\theta}{r}\frac{\partial T}{\partial \theta} = \frac{V_\theta}{a}\frac{\partial T}{\partial \theta} \tag{5}$$

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Substituting Equations 1, 2, and 5 into Equation 3 and introducing the new variable y defined above, yields

$$\frac{\partial T}{\partial \theta} = \frac{\alpha}{\Omega}\frac{\partial^2 T}{\partial y^2} \tag{6}$$

The relevant boundary conditions are

$$T = 0 \text{ at } y = \infty \text{ and } 2\pi > \theta > 0 \tag{7}$$

$$T = T_a \text{ at } y = 0 \text{ and } 2\pi > \theta > 0 \tag{8}$$

The solution of Equation 6 subject to the above conditions is (Carslaw and Jaeger 1959)

$$T = T_a \operatorname{erfc}\left[\frac{y}{2(\alpha\theta/\Omega)^{1/2}}\right] \tag{9}$$

The local convective rate of heat transfer per unit area of the surface of the cylinder is obtained as follows

$$q_\theta = -K\left(\frac{\partial T}{\partial y}\right)_{y=0} = KT_a\left(\frac{\pi\alpha\theta}{\Omega}\right)^{-1/2} \tag{10}$$

If the local heat transfer coefficient h_θ is defined as $q_\theta = h_\theta T_a$, then the local Nusselt number is given by

$$\mathrm{Nu}_\theta = 0.798(\mathrm{RePr}/\theta)^{1/2} \tag{11}$$

where $\mathrm{Re} = (2a)^2\ \Omega/2\nu$ and $\mathrm{Pr} = \nu/\alpha$ are the rotational Reynolds number and Prandtl number, respectively.

The average Nusselt number for the rotating cylinder is obtained from

$$\mathrm{Nu} = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{Nu}_\theta\, d\theta \tag{12}$$

as

$$\mathrm{Nu} = 0.6366(\mathrm{RePr})^{1/2} \tag{13}$$

Verification of the theory

The theory presented above predicts the effects of rotation on the convective heat transfer to an isothermal circular cylinder rotated in a fluid at rest. We compare it with the experimental results of Jones et al. (1988) and the experimental correlation of Dropkin and Carmi (1957). We have done it in Figure 1 where the agreement is remarkable with the present theory. The cylinder

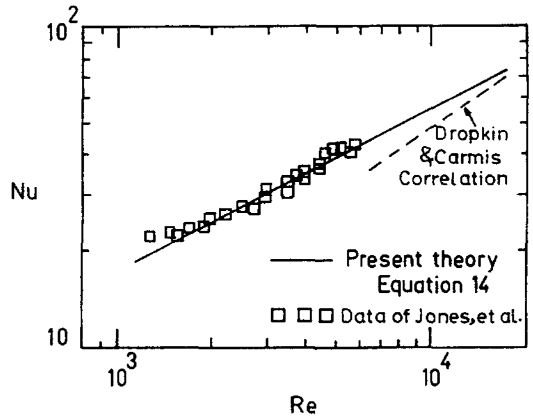

Figure 1. Comparison of the present theory with the experimental results

was rotating in still air ($\mathrm{Pr} = 0.70$) in these experiments; therefore, Equation 13 becomes

$$\mathrm{Nu} = 0.533\ \mathrm{Re}^{1/2} \tag{14}$$

The present solution is expected to give fair agreement with experimental data with fluids of low Prandtl number (e.g., gases and light liquids). The inviscid flow solution of the spinning fluid renders the present approximation suitable for large Reynolds number flows of small viscosity. The present solution is limited to rotational flow with $\mathrm{Re} > 0(10^3)$. This flow limit was indicated by Kays and Bjorklund (1958). Below this limit, heat transfer is predominated by free convection.

Conclusions

The analytical solution obtained in this paper for the convective heat transfer over the surface of a rotating cylinder has been successful in calculating the local and average heat transfer rates. The results given by this solution compared favourably with the available experimental data for fluids of low Prandtl number.

References

Carslaw, H. S. and Jaeger, J. C. 1959 *Conduction of Heat in Solids*, 2nd ed. Clarendon Press, Oxford, 88

Daskalakis, J. 1993 Mixed free and forced convection in the incompress-

Notation

a	radius of the cylinder, *m*
h_θ	local convective heat transfer coefficient, $\mathrm{W/m^2\ K}$
K	thermal conductivity of fluid, W/mK
Nu	average Nusselt number, $h(2a)/K$
Nu_θ	local Nusselt number, $h_\theta(2a)/K$
Pr	Prandtl number, ν/α
q_θ	local heat transfer, $\mathrm{W/m^2}$
Re	rotational Reynolds number, $(2a)^2\Omega/2\nu$
r	radial coordinate, m
T	absolute temperature, K
T_a	absolute temperature at cylinder surface, K
V_r	radial velocity component of the flow, m/s
V_θ	angular velocity component of the flow, m/s
y	radial variable from cylinder surface, m

Greek

α	thermal diffusivity, $\mathrm{m^2/s}$
δ	boundary layer thickness, m
θ	angular coordinate
ν	kinematic viscosity, $\mathrm{m^2/s}$
Ω	cylinder angular velocity, $\mathrm{s^{-1}}$

ible boundary layer along a rotating vertical cylinder with fluid injection. *Int. J. Eng. Sci.*, **17**, 689–695

Dropkin, D. and Carmi, A. 1957. Natural convection heat transfer from a rotating horizontal cylinder rotating in air. *Trans. ASME*, **77**, 741–749

Jones, J., Poulikakos, D. and Orozco, J. 1988 Mixed convection from a rotating horizontal heated cylinder placed in a low-speed wind tunnel. *Int. J. Heat Fluid Flow*, **9**, 165–173

Kays, W. M. and Bjorklund, I. S. 1958 Heat transfer from a rotating cylinder with and without cross flow. *Trans. ASME*, **80**, 70–78

Prandtl, L. and Tietjens, O. 1934. *Hydro and Aerodynamics*, Vol. 2. McGraw-Hill, New York

Schlichting, H. 1979. *Boundary Layer Theory*, 2nd ed. McGraw-Hill, New York, 88